High-intensity Vacuum Ultraviolet Continuum Emission from Intense Relativistic Electron Rings

U. Schumacher

IPP 0/46

Nov.80



# MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

8046 GARCHING BEI MÜNCHEN

# MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

### GARCHING BEI MÜNCHEN

High-intensity Vacuum Ultraviolet Continuum Emission from Intense Relativistic Electron Rings

U. Schumacher

IPP 0/46

Nov.80

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

High-intensity Vacuum Ultraviolet Continuum Emission from Intense Relativistic Electron Rings

### Abstract

Intense relativistic electron rings can emit a high-intensity continuum in the vacuum ultraviolet spectral region if the ring parameter range is moderately extended beyond the values used at present in electron ring accelerators. Although the ring dimensions are very much smaller than those of conventional accelerators and storage rings acting as synchrotron radiation sources, the electron current in the rings can be made very high, being limited only by collective instability thresholds and technical constraints, so that electron rings can afford a relatively cheap source of high-intensity vacuum ultraviolet radiation with a continuous spectrum. The radiation is pulsed with a duration of about 100  $\mu s$ , and even with a repetition rate of about 10 Hz the time-averaged brightness of this source could be higher in the VUV region than that of present storage rings.

#### 1. Introduction

Although synchrotron radiation <sup>1,2</sup> as the main mechanism for energy loss was originally regarded as an undesirable by-product of high-energy particle accelerators, interest in the application of intense vacuum ultraviolet and soft X-rays from circular accelerators for atomic, molecular and solidstate spectroscopy has considerably increased during the last three decades <sup>3-8</sup>.

Charged particles with relativistic velocities in circular accelerators emit synchrotron radiation with very special properties. The radiation has a continuous spectrum from the far infrared into the VUV or even X-ray regions, which, for incoherent emission, can be absolutely calculated from the parameters of the particle beam. The radiation is emitted into a very small angular cone around the instantaneous direction of flight of the particle. The light is linearly polarized in the orbit plane (with the electric vector parallel to it) and elliptically polarized outside this plane.

Synchrotron radiation is of great importance for applications in the vacuum ultraviolet spectral region. In this wavelength range (below the characteristic wavelength) the radiation intensity predominantly depends on the particle current. Since the currents applied in electron rings used for the electron ring accelerator <sup>9,10</sup> are very much higher than in present circular electron and positron accelerators and storage rings, this paper treats the question whether small electron rings with high currents might serve as high-brightness synchrotron radiation sources for the vacuum ultraviolet region.

## 2. Synchrotron Radiation Properties

The angular distribution and the spectrum of synchrotron radiation are characterized by the characteristic cut-off energy <sup>8</sup>

(1a) 
$$\varepsilon_{c}[eV] = 2.218 \times 10^{-4} E^{3} [MeV]/R[cm] = 6.651 \times 10^{-4} B[T] \cdot E^{2} [MeV]$$

or characteristic wavelength

(1b) 
$$\lambda_{c}[R] = 4.189 \times 10^{8} R[cm]/\gamma^{3} = 12400/\epsilon_{c}[eV]$$
.

At the photon energy  $\epsilon=\epsilon_{\rm C}$  the angular spread  $\Delta\psi$  (FWHM) of the radiation intensity is about 1/ $\gamma$  with  $\gamma=E/m_{\rm O}c^2$  and E the particle energy. For smaller photon energies ( $\epsilon<<\epsilon_{\rm C}$ ) the angular spread is approximated by

(1c) 
$$\Delta \psi \simeq \frac{2}{\Upsilon} (\varepsilon_{\rm C}/\varepsilon)^{1/3}$$
.

The spectral distribution  $P_{\lambda}(\lambda)$  of synchrotron radiation strongly depends on the electron energy  $\gamma$ . Numerically, one has for one electron <sup>1</sup>

(2) 
$$P_{\lambda}(\lambda)[W/\Re] = 6.83 \times 10^{-25} \frac{\gamma^7}{R^3[cm]} \cdot \left(\frac{\lambda_c}{\lambda}\right)^3 \int_{C/\lambda}^{\infty} K_{5/3}(\zeta) d\zeta$$

where the numerical values of the quantity

PW = t 
$$\int_{0}^{\infty} K_{5/3}(\zeta) d\zeta$$
 with t =  $\lambda_{c}/\lambda = \varepsilon/\varepsilon_{c}$ 

are obtained from, for example, Laslett  $^{11}$  or van Steenbergen  $^{27}$ .  $K_{5/3}$  is a Bessel function of the second kind.

The spectrum of synchrotron radiation can be roughly divided into two regions:

In the high-energy region above the cut-off energy  $\varepsilon_{_{\mathbf{C}}}$  the intensity sharply drops with  $\varepsilon$ . By increasing the particle energy the spectrum can hence be extended to higher and higher photon energies. In the low-energy region ( $\varepsilon$  <<  $\varepsilon_{_{\mathbf{C}}}$ ) - on which we concentrate at first to facilitate comparison with existing publications - the brightness [photons/(sterad·s·cm²·eV)] of the synchrotron radiation in the plane of the orbit can be approximated by  $^{8}$ 

(3)  $B(O,\varepsilon)$  [photons/(sterad·s·cm<sup>2</sup>·eV)] =

= 
$$1.57 \times 10^{19} R^{2/3} [cm] \cdot \epsilon^{-1/3} [eV] \cdot I_e [A] \cdot A^{-1} [cm^2]$$
.

The synchrotron radiation intensity in the low-energy spectral region hence does not depend on the particle energy E, but on the accelerator radius R, the beam current  $\mathbf{I}_{e}$  and the source area (beam cross-section) A. For present electron and positron accelerators and storage rings the radii R are in the range of several to hundreds of meters, while the currents are of the order of several milliamperes to about a few amperes.

Although in electron ring accelerators the major radii are only in the range of a few centimeters, the currents  $\mathbf{I}_{e}$  can be made very much higher than in present electron accelerators and storage rings.

# 3. Electron Ring Properties for High Brightness Synchrotron Radiation in the VUV

In electron ring accelerator (ERA) experiments synchrotron radiation has often been observed and applied as a diagnostic method  $^{12-15}.$  Because of the relatively low electron energies (E  $\simeq$  15 MeV) the cut-off energies were small and the characteristic wavelengths were in the near infrared region ( $\lambda_{C} \simeq$  10  $\mu m$ ).

The characteristic wavelength can be shifted to shorter values (into the VUV region) by an increase of the electron energy (according to eq.(1b)), which is possible with further electron ring compression. We then have to ask what beam currents  $I_e$  can be obtained to get high brightness of the synchrotron radiation (from eq.(3)) in the VUV spectral region.

There are essential limitations on the electron number of an electron ring circulating in a pulsed axisymmetric high magnetic field. The most important limit is imposed by the condition for stability against collective instabilities, predominantly the negative mass instability, which limits the maximum electron number in the ring to 10,16

$$N_{e} \leq \left(\frac{1}{1-n} - \frac{1}{\gamma^{2}}\right) \frac{\gamma R}{2\beta^{3} r_{O} |Z_{m}/(mZ_{O})|} (\Delta E/E)^{2},$$

where n =  $-\frac{R}{B}\frac{\partial B}{\partial R}$  is the magnetic field index,  $r_{O}$  the classical electron radius,  $|Z_{m}/(mZ_{O})|$  the coupling impedance of the m-th mode related to the impedance of free space  $Z_{O}=377~\Omega$ , and  $\Delta E/E$  the particle relative energy spread. With  $\beta\simeq 1$ ,  $(1-n)^{-1}>> \gamma^{-2}$ , n = 0.5, and using  $|Z_{m}/(mZ_{O})|\simeq 3.18~\sigma_{a}/R$  according to Faltens and Laslett  $^{17}$ , where  $\Delta E/E\simeq 2.36~(1-n)\sigma_{a}/R=1.18~\sigma_{a}/R$ , and  $\sigma_{a}$  is the standard deviation of the radial minor ring dimension, this results in

$$N_e = 1.554 \times 10^{12} \cdot \gamma \cdot R[cm] \cdot \frac{\sigma_a}{R}$$

with the gyration radius ( $\beta \simeq 1$ )

(4) 
$$R[cm] = 0.1704 \cdot \gamma / B[T]$$
.

For the electron current

$$I_{e} = \frac{N_{e}e\beta c}{2\pi R}$$

we hence get

$$I_e[A] = 1.187 \times 10^3 \cdot \gamma \cdot \frac{\sigma_a}{R}$$

or with  $\sigma_a/R = 0.1$ 

(5) 
$$I_e[A] = 118.7 \cdot \gamma$$
.

From eq.(3) we thus obtain for the brightness of the synchrotron radiation from these electron rings

(6) 
$$B(0,\epsilon)[photons/(sterad \cdot s \cdot cm^2 \cdot eV)] =$$

$$= 5.73 \cdot 10^{20} \gamma^{5/3} \cdot B^{-2/3} [T] \cdot \epsilon^{-1/3} [eV] \cdot A^{-1} [cm^2].$$

If we try to keep the major radius R relatively small in order to obtain a small, inexpensive device, we are technically limited to pulsed magnetic field strengths of B  $\simeq$  18 T. (It is, however, not necessary to go to these high values since, as is obvious from eqs.(1) and (4), the high  $\gamma$  values can also be obtained with larger dimensions at smaller field levels.)

With this value an example of electron rings (with  $\gamma=141$  and hence  $\epsilon_{\rm c}=62$  eV,  $\lambda_{\rm c}=200$  Å, and with a minor cross-section of  $\Lambda=0.03$ , as given by the minor radial and axial dimensions as the synchrotron and betatron widths) is chosen, and the dependence of its brightness on the photon energy  $\epsilon$  is plotted in Fig.1, together with the values of DESY and DORIS, as given by Kunz  $^{6,8}$ . For the values near and above  $\epsilon_{\rm c}$  the approximation eq.(3) cannot be applied since then the exact solution  $^{1}$  (according to eq.(2)) has to be used. However, with the approximations for PW  $^{11}$  (which is a universal function for the synchrotron radiation  $^{27}$ )

$$PW = t \int_{0}^{\infty} K_{5/3}(\eta) d\eta \simeq \begin{cases} 2\Gamma\left(\frac{2}{3}\right)\left(\frac{t}{2}\right)^{1/3} & \text{for } t = \frac{\varepsilon}{\varepsilon_{C}} << 1 \\ \sqrt{\frac{\pi}{2}} t \exp(-t) & \text{for } t = \frac{\varepsilon}{\varepsilon_{C}} >> 1 \end{cases}$$

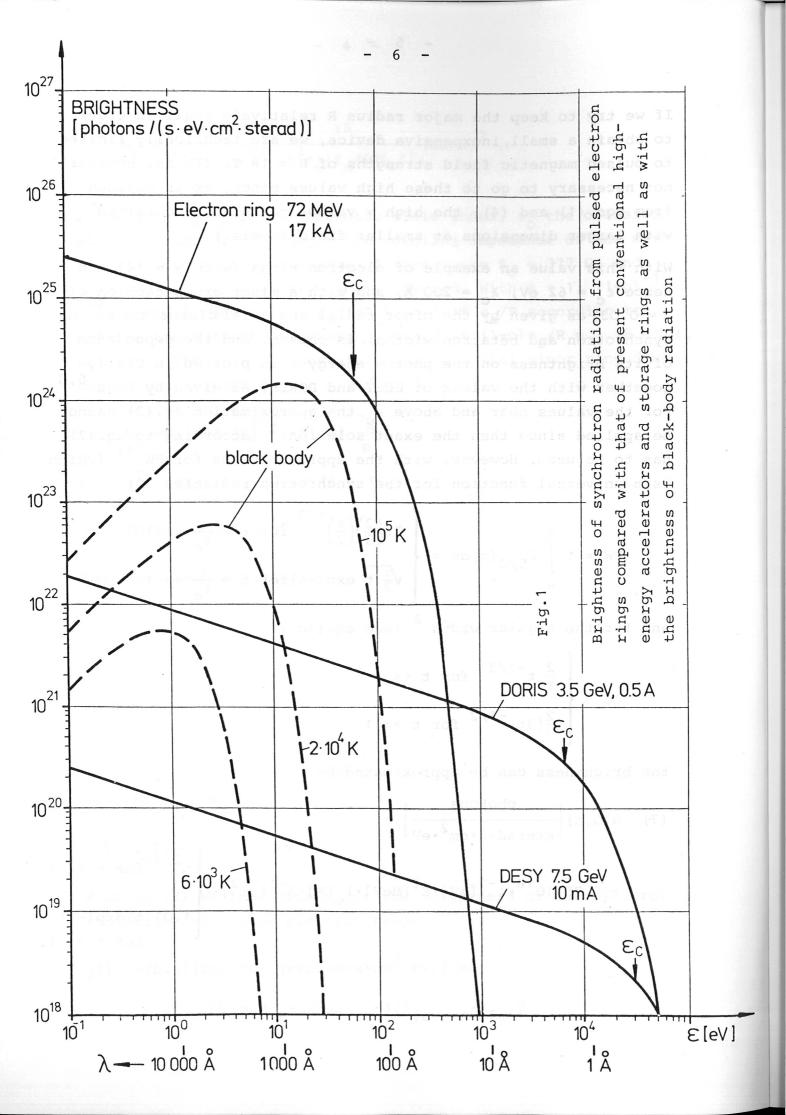
and for the angular width 8 (see eq.(1c))

$$\Delta \psi \simeq \begin{cases} \frac{2}{\gamma} t^{-1/3} & \text{for } t << 1 \\ \frac{2}{\gamma} (3t)^{-1/2} & \text{for } t >> 1 \end{cases}$$

the brightness can be approximated by

(7) 
$$B(O,\varepsilon)\left[\frac{\text{photons}}{\text{sterad}\cdot\text{s}\cdot\text{cm}^2\cdot\text{eV}}\right] \simeq$$

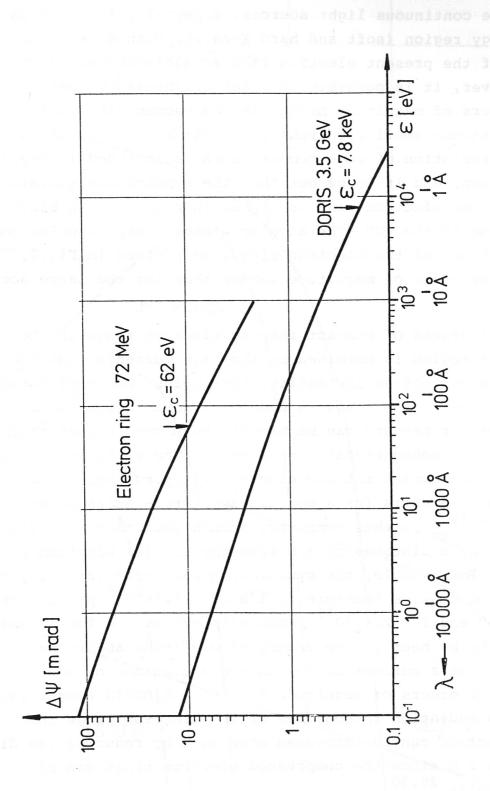
$$\simeq 5.75\times10^{16}\cdot\varepsilon^{-1}[\text{eV}]\cdot\text{E}^2[\text{MeV}]\cdot\text{I}_{\text{e}}[\text{A}]\cdot\text{A}^{-1}[\text{cm}^2]\cdot\begin{cases} t^{2/3} \text{ for } t << 1\\ 1.01 \text{ t } \exp(-t) \text{ for } t >> 1.\end{cases}$$



We conclude from Fig.1 that high-energy electron and positron accelerators and storage rings (such as DESY and DORIS) are very intense continuous light sources, especially in the high photon energy region (soft and hard X-rays). With a moderate extension of the present electron ring accelerator parameter range, however, it is possible to reach brightness values several orders of magnitude higher in the vacuum ultraviolet region by intense small electron rings. The brightness of the black-body radiation of some temperatures is included in Fig.1 for comparison, and it turns out that the synchrotron radiation 10<sup>5</sup>K blackintensity of an electron ring is higher than the body emission in the VUV. The angular widths (perpendicular to the orbit plane) of the electron rings, as plotted in Fig. 2, are about one order of magnitude larger than for the large accelerators.

The high brightness of compact intense electron rings in the VUV spectral region is obtained by the high currents that are close to the collective instability thresholds. For applications the compactness of the rings is even more advantageous since the spectrometers or targets can be brought very much closer to the source than for conventional accelerators. The ratio of the VUV intensities of compact intense electron rings to those of conventional accelerators for a certain aperture - as is often plotted 6,18,19 - is thus even very much higher than the brightness ratio. At a distance of 2 m from the  $\gamma$  = 141 electron ring (of Fig.1), for example, the synchrotron radiation intensity into a 2 cm  $\times$  2 cm wide aperture will be I  $\simeq$  1.5 $\times$ 10<sup>19</sup> photons/(s·eV) at  $\epsilon = 10 \text{ eV}$  and I  $\simeq 2.4 \times 10^{-8}$  photons/(s·eV) at  $\epsilon = 100 \text{ eV}$  and hence will be by nearly five orders of magnitude above the DORIS intensities (at a current of 500 mA and a distance of 40 m) 6, or about four orders of magnitude for the 6 A DORIS rings. The synchrotron radiation intensities of the electron rings into a given aperture can be increased even more by reducing the distance below 2 m since the compressed electron rings can be shifted axially 29,30

The synchrotron radiation intensity of the rings is not constant in time since the rings lose radiation energy. The total synchrotron radiation power emitted is



Angular spread of the synchrotron radiation of an electron ring compared with that of a present storage ring Fig.2

$$P[W] = 8.85 \times 10^{-6} E^{4} [MeV] \cdot I_{e}[A] / R[cm]$$

which gives about P  $\simeq$  3 MW for the stated example of  $\gamma$  = 141 electron rings in a 18 T magnetic field with an index of n = 0.5. The characteristic time

$$\tau_{syn} = E/(dE/dt)$$

or, numerically,

$$\tau_{\text{syn}}[s] = 23.7 \cdot R^2 [\text{cm}] / E^3 [\text{MeV}]$$

is about  $\tau_{\text{syn}}$  = 120  $\mu \text{s}$  for the  $\gamma$  = 141 rings. Owing to the strong radiation the energy of the electrons changes relatively fast. In the spectral region below the characteristic energy  $\epsilon_{c}$ , however, the synchrotron radiation intensity remains constant for an appreciable fraction of this time since for  $\epsilon <<$   $\epsilon_{_{\rm C}}$  the brightness does not depend on the electron energy, which is obvious from eq.(3) (see also Kunz 6). Because of its quadratic dependence on the electron energy E the characteristic energy  $\epsilon_{c}$  drops fast. As soon as it becomes smaller than  $\epsilon$ , the intensity of the synchrotron radiation drops appreciably. Figure 3 illustrates this time behaviour for the electron ring parameters of Fig.1. The electron energy E and, even more, the characteristic energy  $\epsilon_{_{\mathbf{C}}}$ drop fast as functions of time. However, the brightness for the photon energy  $\epsilon$  = 10 eV, which is much smaller than the initial critical photon energy  $\epsilon_{c}$  = 62 eV, stays about constant for as much as nearly 150 µs. The brightness for higher photon energies, as in the example of  $\varepsilon$  = 50 eV, decreases rapidly with time. During this time, however, the radius changes appreciably, which is neglected here, as is the damping of the momentum spread  $^{28}$ . This momentum spread, which is necessary for collective instability suppression, results in a brightness reduction 15 only if  $\epsilon_{_{\mathbf{C}}}$  <  $\epsilon$  and hence gives rise to a further small narrowing of the light pulse length in the high  $\epsilon$  case, which is not included in the figure either.

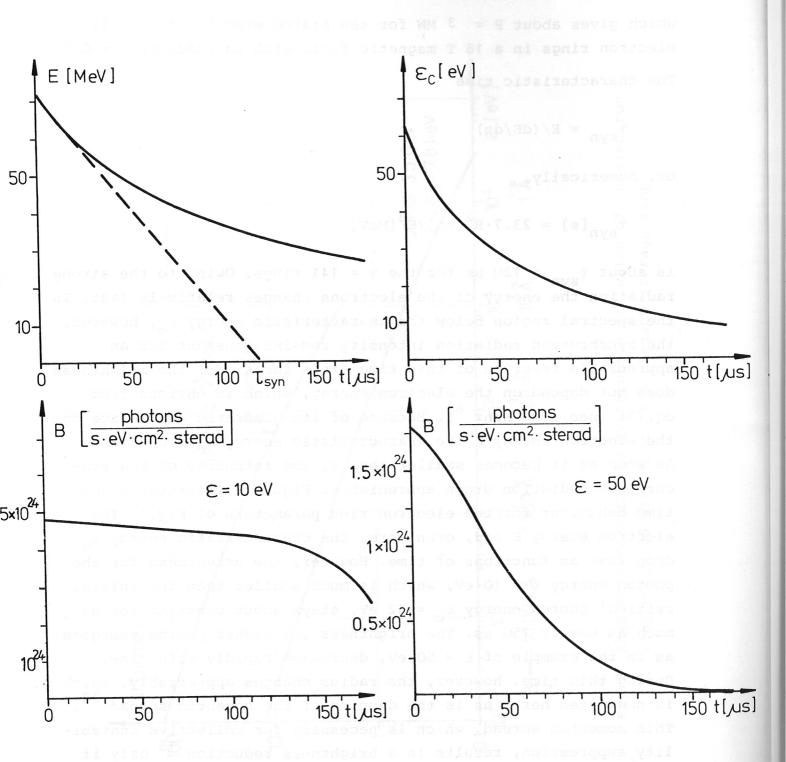


Fig.3 Time dependence of electron energy E, critical photon energy  $\epsilon_{_{\hbox{\scriptsize C}}}$ , and brightness B at two different photon energies for the example in Fig.1

The synchrotron radiation source in the VUV spectral region hence will be a pulsed source with a pulse length of the order of 50 to 100  $\mu s$ . In order to approximate a quasi-continuous light source, one should aim at a relatively high repetition rate. With a repetition rate of 10 to 20 Hz the duty cycle of this quasi-continuous light source would be about  $10^{-3}$ , so that the time-averaged radiation brightness still would be up to one order of magnitude above the values of present storage rings.

### 4. Generation of Intense Electron Rings

Since the characteristic time of the electron rings is relatively small and we do not provide for electric acceleration (with cavities) to compensate the radiation loss, the ring generation (at least the last part of the ring compression) has to be performed on a time scale of a few microseconds. This fast pulsed technology for production and compression of electron rings was applied in the Garching ERA experiments <sup>13,20</sup>. Since the device is limited in space, the costs are relatively low, as for the electron gun (particle energy of about 2 MeV and current of a few kA) with its high efficiency. In the example mentioned we would start with an electron beam with an energy of about 2 MeV and a current of several kA in a magnetic field of about 10 mT to form rings of about 0.5 to 1 m in radius. In successively pulsed field coils the rings would be compressed down to a radius of about 1.3 cm. According to eq.(4) the electron ring compression to increase the electron energy can, however, also be performed at larger radii and hence smaller magnetic field values.

The electron beams with the necessary instantaneous energy spread can be generated  $^{21}$ . At the end of compression the requirements of a medium field index (n  $\approx$  0.5) and low coupling impedance can be met by means of conducting axisymmetric structures ("n-spoilers") on the axis  $^{20}$ . It might, however, be impossible to fulfil the conditions for collective instability suppression during all the compression for the high electron ring currents. If a ring current one order of magnitude lower could be accepted, the obtainable brightness in the VUV region or the intensity in a certain aperture would still be about two or four orders of magnitude, respectively, above those of conventional accelerators.

## 5. Possibilities for Intensity Increase

Up to now we have only treated incoherent emission of synchrotron radiation, this assumption seemingly being confirmed by the experiments <sup>12,13</sup>. At higher electron densities the intensity might, however, be dramatically increased by coherent emission proportional to the square of the density. Such coherent emission - at very low frequencies - was observed with electron rings and is apparently due to collective instabilities <sup>22</sup>. If at maximum compression of the electron rings we had, for instance, resonance behavior of the coupling impedance, strong coherent radiation would occur.

Another possibility to increase the synchrotron radiation intensity, especially in the VUV region, is by magnetic wavelength shifting 23 (undulators 24 or wigglers 25). This could be realized in pulsed electron ring devices by conducting structures on the axis with azimuthally periodic radius variations, as schematically shown in Fig. 4. At the end of compression the electron ring is in a region axially bounded by two periodically slotted disks that - owing to the image currents - contribute to the formation of periodic magnetic field variations at the electron ring location. In order to avoid high coupling impedances, the ring is kept relatively close to a conducting rod on the axis, the axial shape of which, moreover, serves for obtaining the desired high magnetic field index. On the other hand, the rod on the axis might carry an axial current such as to produce an azimuthal magnetic field  $B_{0}$  which proves advantageous for collective instability suppression <sup>26</sup>. Numerical calculations to optimize the structures and the related ring compression cycle have yet to be performed.

Moreover, the synchrotron radiation intensity through a certain aperture can be enhanced by a large factor using suitable mirror systems  $^{31,32}$ . In this case there is no limitation with respect to time distortion as in the "magic mirror" arrangement  $^{8,32}$ .

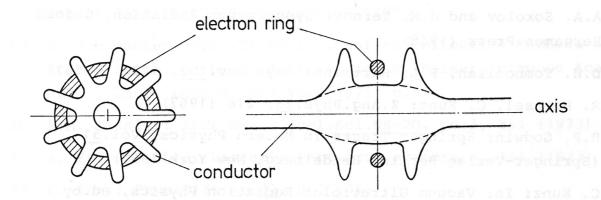


Fig. 4 Schematic of a passive undulator for pulsed electron rings

### Acknowledgement

The author expresses his gratitude to C. Andelfinger, W. Engelhardt, G. Fußmann, W. Herrmann, H. Röhr, and A. Schlüter for fruitful discussions. He is indebted to J.M. Peterson, A. Faltens, and D. Keefe (LBL Berkeley) for helpful comments.

### References

- 1 J. Schwinger: Phys.Rev.75, 1912 (1949)
- 2 A.A. Sokolov and J.M. Ternov: Synchrotron Radiation, Oxford Pergamon Press (1968)
- 3 D.H. Tomboulian, P.L. Hartmann: Phys. Rev. 102, 1423 (1956)
- 4 R. Haensel, C. Kunz: Z.Ang.Phys. 23, 276 (1967)
- 5 R.P. Godwin: Springer Tracts in Modern Physics, Vol.<u>51</u> (Springer-Verlag Berlin, Heidelberg, New York 1969)
- 6 C. Kunz: In: Vacuum Ultraviolet Radiation Physics, ed.by E.E. Koch, R. Haensel and C. Kunz (Pergamon-Vieweg, Braunschweig 1974) p.753
- 7 Synchrotron Radiation and New Developments, ed.by F. Wuilleumier and Y. Farge, Nucl.Instr.and Meth. 152 (North-Holland, Amsterdam 1978)
- 8 Synchrotron Radiation, Techniques and Applications, ed.by C. Kunz, Topics in Current Physics, Vol. 10 (Springer-Verlag Berlin, Heidelberg, New York 1979)
- 9 G.V. Dolbilov, V.I. Mironov, V.G. Novikov, E.A. Perelshtein, G.K. Radonov, V.P. Sarantsev, A.P. Sumbaev, S.I. Tutunnikov, V.P. Fartushny, A.A. Fateev, A.S. Schuelin: Proc.Third Int. Conf.on Collective Methods of Acceleration, ed.by N. Rostoker and M. Reiser, Harwood Acad.Publ., Chur, London, New York 1979, p.83
- 10 U. Schumacher: Springer Tracts in Modern Physics, Vol. 84, p.145-231 (Springer-Verlag Berlin, Heidelberg, New York 1979)
- 11 L.J. Laslett: LRL Berkeley Report ERAN-142 (1971)
- D. Keefe, G.R. Lambertson, L.J. Laslett, W.A. Perkins, J.M. Peterson, A.M. Sessler, R.W. Allison, W.W. Chupp, A.U. Luccio, J.B. Rechen: Phys.Rev.Lett. 22, 558 (1969)
- 13 C. Andelfinger, W. Herrmann, A. Schlüter, U. Schumacher, M. Ulrich: IEEE Trans.Nucl.Sci.NS-20, No.3, 790 (1973)

- 14 V.P. Sarantsev, V.S. Alexandrov, L.S. Barabash, G.V. Dolbilov, V.I. Mironov, V.G. Novikov, G.K. Radonov, A.P. Sumbayev, S.I. Tyutyunikov, B.P. Fartushney, A.A. Fateyev, A.S. Sheulin: Report JINR <u>P9-10917</u>, Dubna/USSR (1977)
- 15 U. Schumacher: Proc.Third Int.Conf.on Collective Methods of Acceleration, ed.by N. Rostoker and M. Reiser, Harwood Acad. Publ., Chur, London, New York 1979, p.71
- 16 A.M. Sessler: IEEE Trans. Nucl. Sci. NS-20, No. 3, 854 (1973)
- 17 A. Faltens, L.J. Laslett: Part.Accelerators 4, 151 (1973)
- 18 C. Kunz: Phys.Blätter 32, 9 (1976)
- 19 F. Wuilleumier: LURE Report 74/03, Orsay (1974)
- 20 C. Andelfinger, W. Dommaschk, I. Hofmann, P. Merkel, U. Schumacher, M. Ulrich: Proc.9th Int.Conf.on High Energy Accelerators, Stanford/USA, p.218 (1974)
- 21 J. Fink, H.-B. Schilling, U. Schumacher: J.Appl.Phys. <u>51</u>, 2995 (1980)
- 22 J. Fink, W. Herrmann, W. Ott, J.M. Peterson: Proc.9th Int. Conf. on High Energy Accelerators, Stanford/USA, p.223 (1974)
- 23 E.M. Rowe, F.E. Mills: Part.Accelerators 4, 211 (1973)
- 24 A. Hofmann: Ref.7, p.17
- 25 H. Winick, T. Knight (eds.): Wiggler magnets, Stanford/USA, SSRP Report 77/05 (1977)
- 26 L.J. Laslett, U. Schumacher: Proc.VIIIth Int.Conf.on High Energy Accelerators, CERN, Geneva (1971), p.468
- 27 A. van Steenbergen: IEEE Trans.Nucl.Sci.NS-26, No.3, 3785 (1979)
- 28 J.M. Hauptman, L.J. Laslett, W.W. Chupp, D. Keefe: Proc. IXth Int.Conf.on High Energy Accelerators, Stanford/USA, p.240 (1974)
- 29 C. Andelfinger, E. Buchelt, W. Dommaschk, J. Fink, W. Herrmann, I. Hofmann, A. Luccio, P. Merkel, D. Jacobi, H.-B. Schilling, A. Schlüter, U. Schumacher, M. Ulrich: IEEE Trans.Nucl.Sci.NS-24, No.3,1622 (1977)

- 30 W. Herrmann: Proc. II. Symp.on Collective Methods of Acceleration, Dubna 1976, p.60
- 31 D. Keefe: unpublished, communicated in a letter by J.M. Peterson
- 32 R. Lopez-Delgado, H. Szwarc: Opt. Comm. 19, 286 (1976)